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PROPORTIONAL REASONING INSTRUCTION: A COGNITIVELY GUIDED APPROACH TO PROFESSIONAL DEVELOPMENT

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Content-specific cognitively guided instruction (CGI) professional development programs have been shown to lead to positive changes in instruction in the elementary grades. This paper presents the results of a study that investigates how teachers use new knowledge gained from a CGI professional development workshop, on proportional reasoning in the middle grades, to inform their instructional decisions. Four teachers' instruction and their rationales for their instructional decisions were examined before and after the workshop intervention. All four teachers' instruction changed to become more cognitively guided.

Introduction

The National Council of Teachers of Mathematics [NCTM] (2000) recommends that proportional reasoning become a major focus of the middle grades curriculum by interweaving it with percents, similarity, scaling, linear equations, graphs, probability, histograms, and problem solving. Furthermore, proportionality is involved in at least one of the focal points for each of the middle grades. Students who fail to develop an understanding of proportional reasoning in middle school are likely to struggle in algebra and other higher level mathematics (Langrall & Swafford, 2000).

Although proportional reasoning and facility with rational numbers are so important, research shows that many students struggle with them (Misailidou & Williams, 2003; Singh, 2000). In fact, preservice and inservice teachers often lack a deep understanding of proportionality (Bezuk, 1988; Conner, Harel, & Behr, 1988). Furthermore, teachers often have weak pedagogical content knowledge (Hines & McMahon, 2005) and knowledge of their students (Ruchti, 2005) related to proportional reasoning.

In order to understand the true breadth of the area of proportional reasoning, teachers must understand both the subtle (e.g. number structure) and obvious (e.g. context) nuances of proportion problems (Franke, Fennema, & Carpenter, 1997). Additionally, teachers need to be able to analyze a problem to determine the range of strategies that students are likely use when solving it. This ability better enables them to determine the difficulty level of a problem. According to Carpenter and Fennema (1991), "analyzing problems, strategies, and development allows for the ability to select critical problems both to differentiate between levels of performance but also to help children progress to the next level" (p. 9).

In 1989, Carpenter, Fennema, Peterson, Chiang, and Loef developed an approach to professional development, cognitively guided instruction (CGI), which focused on increasing teachers' pedagogical content knowledge and knowledge of their students in a specific content area. Since, participation in a content-specific CGI program has been shown to have a positive effect on teachers' pedagogical content knowledge and lead to changes in classroom practice towards instruction guided by students' thinking.

This study investigates how teachers use new knowledge gained from a CGI professional development program on proportional reasoning to inform their instructional decisions. Particularly, the ways that the teachers generated student thinking prior to and after gaining access to research-based models of students' thinking about proportional reasoning were examined. This study is intended to extend the CGI studies, which focused primarily on operations in the elementary grades, to proportional reasoning in the middle grades.

Theoretical Framework

A few assumptions about the goals of instruction and what instruction should look like underlie the CGI philosophy (Carpenter & Fennema, 1991). First, the goal of instruction is to increase students' understanding by connecting to their prior knowledge (Donovan & Bransford, 2005). To effectively structure instruction to build off students' prior knowledge and experiences, teachers must have strong pedagogical content knowledge, as well as a deep understanding of their students.

Second, effective instruction requires teachers to make informed instructional decisions, by continually reflecting on their instruction and students' learning. Thus, teachers must continually assess their students' understandings through using appropriate tasks and questions that enable students to showcase their thinking.

Third, for learning to occur students need to develop mathematical explanations and justifications and connect them to their existing body of knowledge. The teacher's role is to choose meaningful exploration activities and provide a learning environment where mathematical conjectures, multiple techniques, and mathematical dialog are valued.

Methodology

The professional development program which provides the context for this study is briefly described here. Data collection and analysis methods will also be discussed.

Professional Development Program

The context of this study was a two-day professional development workshop, followed by ongoing implementation support over the six subsequent months. The workshop design was inspired by the CGI studies. During the workshop, we discussed the research findings in the area of proportional reasoning that are related to (a) students' strategies; (b) problem types; (c) factors influencing students' success and strategy choices; (d) prerequisites to the development of proportional reasoning; and (e) developmental theories, specifically those of Piaget (Piaget & Inhelder, 1969), Noelting (1980a, 1980b), Milsailidou and Williams (2003), Lesh, Behr, and Post (1987) and Karplus et al. (1983a).

Written and video cases illustrating real classroom teaching episodes were also used to illustrate CGI. The teachers were asked to analyze the pedagogy, questioning, student thinking, and teacher's role within the cases.

Researchers have found that teachers are more likely to change their instruction as a result of professional development when supported (Steele, 2001). Here, when initiated by the teachers, the researcher served as a mentor to aid in planning and analyzing students' strategies.

Participants

The participants in this study, who volunteered, were four middle school teachers from the same school in a rural part of Virginia. The teachers had varying levels of experience from 1 to 22 years. There was one teacher from each grade, 6-8, and one who taught in eighth grade for the first half of this study and seventh grade for the second. The sixth grade teacher (Bob), seventh grade teacher (Abby), and the seventh/eighth grade teacher (Julie) were all in their first year teaching mathematics. However, Julie had four years of science teaching experience. The eighth grade, and veteran, teacher (Sue) had taught mathematics for 22 years. All of the mathematics teachers in the school participated in the workshop.

Data Sources

The data analyzed here were collected primarily through classroom observations, interviews, and document compilation. Each participant was observed and interviewed prior to the workshop to examine their instructional style and bases for making instructional decisions. In addition, documents were collected that illustrated how each teacher had taught or planned to teach proportion concepts.

After the workshop, each teacher was observed teaching ratio and proportion concepts on several occasions, ranging from 4 to 16 instances. The large variation in the number of observations per teacher was due to differences in the depth of curriculum coverage of proportions between sixth and eighth grade. Detailed field notes were compiled during each observation which focused on (a) what the teachers *did* and *said* during instruction, (b) how the students reacted to the instruction, and (c) how the teacher interacted with the students.

Following each observation, the teachers were interviewed by the researcher to determine their rationales for their instructional decisions. The interviews were recorded and later transcribed to capture the teachers' responses accurately. These interviews were semi-structured; they began with a predetermined list of questions, and appropriate follow up questions, and ended with questions related specifically to the observations.

Data Analysis

A case study and cross-case analysis was conducted to describe each teacher's instruction and rationale before and after the workshop. Systematic data analysis was used to derive causal descriptions and lawful relationships among the data. The three frameworks that guided the initial coding were derived from: (a) the content of the CGI workshop on proportional reasoning, (b) Franke, Carpenter, Levi, and Fennema's (2001) schema for analyzing teachers' actions on a CGI scale, and (c) Stein, Smith, Henningsen, and Silver's (2000) schema defining levels of cognitive demand. To ensure validity, the data was triangulated across methods, member checking was used, and colleagues of the author were asked to evaluate the author's interpretations of the data.

Findings

The purpose of this study was to answer the following research questions: (1) How did the four teachers generate student thinking through their instructional decisions *prior* to participating in the workshop? (2) What were the four teachers' instructional decisions based on prior to the workshop? (3) How did the four teachers change in order to generate student thinking through their instructional decisions after participating in the workshop?

How Was Student Thinking Generated through the Teachers' Instructional Decisions Prior to the Workshop?

All four teachers involved their students in their lessons to varying degrees before the workshop, but they all focused their tasks and questions on applying and remembering procedures rather than developing connections and understandings. Thus, the teachers rarely generated student thinking through their instructional decisions.

All four teachers used questioning to involve their students, but all their questions were categorized as placing low cognitive demands on students. For example, the teachers commonly asked students to: state the next step when applying a procedure, give an answer, or perform a computation. Two teachers were observed presenting a task with a high-level of cognitive demand, but their treatment of the task had the effects of lowering the cognitive demand. For example, one teacher had "tricky" problems, which were different than those the students had seen before, at the end of a worksheet. Those tasks were initially categorized as high-level tasks because the students need to form connections or "do mathematics" (Stein et al., 2000). However, the teacher immediately pointed out the tasks as "tricky" and prescribed another procedure for solving them, ultimately lowering the demands of the tasks to "procedures without connections".

What Were the Four Teachers' Instructional Decisions Based on Prior to the Workshop?

Sources. Prior to the workshop, the novice teachers relied on others for the majority of their lesson ideas: two teachers almost exclusively relied on their textbook while the other relied on worksheets and materials provided by colleagues. Unlike the novice teachers, the veteran teacher was not entirely dependent on other sources for lesson ideas.

The novice teachers lacked confidence in their ability to plan appropriate lessons to meet the Virginia Standard of Learning (SOL) objectives, which likely stemmed from their lack of experience teaching mathematics and their unfamiliarity with the content and pedagogical techniques for teaching. One teacher exclusively used the materials that came with the textbook. The role of the textbook was also significant for the veteran teacher. However, she created her own example problems to relate the new concepts to her students' background knowledge and to be relevant to their lives.

Rationales. All four teachers followed the general sequencing determined by their pacing guides or the textbook, but the three novice teachers did so blindly. The three novice teachers did not have an overview of the general sequencing of their courses, indicating that no consideration was given to the order in which they introduced different mathematics topics. One admitted that she planned her lessons one day or one week at a time and followed the pacing guide. When I asked her how she typically planned her lessons, she replied, "Day-by-day. We have our planning guides, they are really helpful. It tells you day-by-day exactly what they need to know."

All four teachers' planning decisions regarding which types of tasks to include in their lessons, as well as how to sequence them, were influenced largely by what appeared in their lesson planning sources and by their goal to have students memorize and master procedures to prepare for state-mandated testing. However, the veteran teacher referenced what her students knew or where they were in their development of understandings when describing her reasons for her instructional decisions, while the other teachers did not.

How did the Four Teachers Change in order to Generate Student Thinking Through Their Instructional Decisions after Participating in the Workshop?

Student involvement and questioning. All four teachers changed, to varying degrees, to encourage more individual high-level thinking from their students. They did so by asking students to solve problems rather than exercises exclusively and by asking high-level questions in addition to low-level questions. They all asked more questions and used more tasks that I categorized as requiring high-levels of cognitive demand; unlike prior to the workshop, when all four teachers posed only low-level questions and tasks. All four teachers allowed their students to solve problems without any prior instruction on how to do so, but Abby and Bob always presented procedures after students were given the opportunity to solve the problems on their own. Prior to the workshop, none of the teachers allowed their students to solve problems without prior instruction.

Sources. The sources that Sue, the veteran teacher, and Bob used for lesson ideas only changed minimally, while the sources the other two teachers used changed more substantially. The novice teachers all still relied on the pacing guide to determine their overall instructional sequence of topics, while the veteran teacher was observed referring to the textbook for the general sequence of topics.

Immediately after the workshop, Julie became less reliant on the textbook materials. She began creating her own warm-up exercises to meet her instructional goals and to highlight her students' relevant background knowledge. Although she initially depended on the textbook to provide her with appropriately sequenced problems, she chose which problems to include in her lessons. In her second time teaching proportion concepts, after moving to the seventh grade, she relied even less on the textbook. Instead, she used what she learned in the workshop, together with problems from a SOL Test Bank, to plan her lessons.

In Abby's case, there was a substantial change in her planning sources. She crafted her own problems for worksheets on proportions, percents, and cross multiplication where before she only used worksheets others had created.

Similar to before the workshop, all the novice teachers said that the overall sequencing of their lessons was determined by the outlined sequence and timing given in their pacing guides. They did not seem to have an overview of the topics and sequence of their courses. They planned their lessons on a weekly basis, often without looking ahead in their pacing guides. There were occasions when they could not tell me what was coming up next or generally when they would be teaching a specific topic.

Rationales. After the workshop, three of the teachers expressed that they had sequenced tasks in order to further their students' development. Of these three teachers, the increase in Julie's attention to the sequencing of the tasks used was the most profound. The fourth teacher, Bob, generally articulated no particular reason for the sequences of tasks he was observed presenting, besides following the sequence in the textbook. However, all four gave some consideration to the numerical structure of the tasks.

Changes in the teachers' levels on the CGI scale. All the teachers increased at least one level according to Franke and colleague's (2001) CGI schema indicating that their instruction was more cognitively guided, after the workshop.

Julie had the most significant changes in her instruction and beliefs, from Level 1 to Level 3 or Level 4A. In Franke and colleague's (2001) schema, Level 1 corresponds to instruction that is not cognitively guided and Level 4B corresponds to instruction that is completely cognitively guided. Julie was at Level 1, prior to the workshop, for several reasons. First, she believed that her students were not capable of solving problems without the help of delineated procedures or their textbooks. During my observation, she did not allow her students to figure out how to solve problems on their own and from her description of her standard lesson plan, it was clear that Julie did not allow her students to invent their own strategies for solving problems. In fact, she never posed a "problem," instead she used tasks where procedures were given or exercises that were similar to previously solved tasks. Like the tasks Julie used prior to the workshop, the questions she asked her students required low-level cognitive demands of students; she did not ask them about their thinking.

After participating in the workshop, Julie satisfied all of the characteristics of a teacher at Level 3. First, while there was no change in Julie's expressed beliefs about how students learn mathematics best, her teaching style changed to corroborate these beliefs. In both instances she indicated that discovery was the best method for learning and that teachers should always allow their students to solve problems, but Julie commented that she learned how to teach in this way through the workshop. Post workshop, Julie *never* taught a procedure before allowing her students to discover their own ways first; in fact, in most cases she avoided teaching procedures all together.

Second, after the workshop, Julie encouraged her students to solve problems in different ways. She asked, "Did anyone do it differently?" so frequently that her students began to volunteer to share alternative approaches. Third, Julie used a variety of different types of problems after the workshop. She attempted to teach in a developmentally appropriate way by creating progressions of problems according to their difficulty levels or the strategies they were likely to elicit from her students. For instance, one of her lessons on proportions progressed from problems involving an integer factor of change to problems without integer factors of change and with large numbers, to encourage her students to first develop a factor of change strategy and then advance to cross multiplication (CM). She said, "I deliberately gave them a problem where factoring was going to blow their minds so that they would be forced into the other [developing CM]." She explained that she "wanted to lead them [her students] into discovering CM or needing the CM algorithm."

Fourth, Julie expected her students to share their thinking with the class, after the workshop. *Before* the workshop, the only questions Julie was observed asking had low-level demands, but *after* the workshop Julie also asked questions with high-level demands. In fact, as more time passed after the conclusion of the workshop, Julie asked more and more questions placing high-level demands on students. Immediately after the workshop, Julie asked her students to share their solution strategies with the class, but she focused more on the processes than on her students' reasons for their actions or why their strategies worked. Over time she asked more questions about her students' thinking, such as, "Why does this work?" or "Why did you put the numbers there [within a proportion]?" Julie's eighth-grade students really responded well to their new responsibilities involving solving problems and sharing their thinking. One of her students went to the board and announced, "All eyes on me. Ok…" and then she proceeded to explain her solution process. This was a drastic change; before the workshop, Julie's students were far more disruptive than participatory. Julie noticed this too and she attributed the change in her students to requiring them to *think*:

> Yeah ever since I switched it [switched her teaching to problem based rather than read in the book and fill in the blanks], he has been into it. Because he is using his mind. And you know, S [another student] did too [got into the lesson].

For these reasons, Julie was at least at Level 3 on the CGI scale after the workshop. But Julie also possessed some of the characteristics of a teacher at Level 4A. Julie may have been in transition to this level. She focused her lessons around problem solving. The majority of the tasks Julie posed were word problems after the workshop. She always asked her students to solve problems in their own ways and to share their approaches with her class. As I mentioned earlier, she based her instructional decisions on her students thinking, attempting to push them further along in their development of understandings. There was also at least one instance when she based an instructional decision during class on her students' thinking. This instance was when one of her students explained that he was writing percent proportions in the order in which the values appeared in the problem. In response, Julie posed a problem where this students' misconception would be highlighted. Later she explained to me, "They wrote it [a student said, 'It has to go 78 over 58 because it is 78% of 58'] and I thought I have to use the other [another] problem to show that it may not go in order."

Sue changed her instruction from Level 2 to Level 3. Prior to the workshop, Sue was at Level 2 for several reasons. Unlike the other teachers, Sue occasionally asked her students to solve problems on their own and repeatedly stressed the importance of relating new knowledge to students' background knowledge (e.g. "If they can tie it to something that they have learned previously, it tends to stick a lot better."). She also described being open to a variety of approaches to solving problems, which was characteristic of a teacher at Level 2. However, that behavior was not observed through her teaching. Because she showed her students how to solve problems with her anticipatory set, note-taking guides, and approach to dealing with students' difficulties, Sue was not at Level 3 prior to the workshop.

After the workshop, Sue's instruction changed in several ways to become more cognitively guided. The following changes led to a Level 3 characterization: (a) Unlike before, Sue allowed her students to solve problems in their own ways without first asking them to read about procedures for solving similar problems. (b) When her students had trouble figuring out a way to solve a novel problem, Sue posed questions to help them discover a solution strategy. She did not resort to dictating a strategy, like she did prior to the workshop. (c) She demonstrated the openness to a variety of strategies that she expressed previously. Her students also perceived her receptivity to different approaches; During one of my observations, Sue asked her students, "Do you have to do it that way?" and one student replied in a mimicking tone, "No, whichever way works for you." (d) Sue asked questions placing higher-level cognitive demands on her students, such as, "Why can we do that?" or "What is another way you could do this?" These types of questions encouraged her students to discuss their thinking, as well as develop connections and understandings.

Both Abby and Bob progressed from Level 1 to Level 2. Prior to the workshop, neither teacher allowed their students to solve problems on their own. Instead, they demonstrated how to solve problems and then asked their students to solve similar exercises. For this reason, it appears that they did not believe that their students were capable of figuring out their own ways to solve problems.

Both teachers were not observed posing questions or tasks that placed high-level demands on their students prior to the workshop. Actually, Abby and Bob used tasks and questions that could have been deemed high-level, but their treatment of them had the effect of lowering the tasks' cognitive demands. Neither Abby nor Bob asked their students about their thinking.

After the workshop, both teachers progressed to Level 2 on the CGI scale. They occasionally allowed their students to solve problems on their own, which they did not do previously. However, they both always taught procedures for solving problems after their students had been given a chance to solve them on their own. Bob recognized that there were a variety of proportion problem types and that those types can have an effect on students' thinking. He varied the types of problems he included on his proportion assessment and spoke about his decision being influenced by the fact that different numerical structures would encourage different strategies. Abby also considered different problem types and varied the numerical structures and contexts she chose to include in her lessons. However, she only allowed her students to use a factor of change or CM strategy. Additionally, both teachers elicited more thinking from their students after the workshop and asked some questions with high-levels of cognitive demand. Abby and Bob were not at Level 3 due to the fact that neither teacher freely allowed their students to solve problems in their own ways.

Conclusions

A content specific professional development program, regardless of content or grade-level, can lead to positive changes in teachers' instruction to become more cognitively guided. Although this workshop intervention was brief, all four teachers changed their instruction to become more cognitively guided afterwards. It is reasonable to assume that a longer professional development opportunity may have lead to greater positive changes in instruction.

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